

## The Case of Mary and Tamara: Shifting Understanding from One Third of a Discrete Whole to One Third of a Group

*The students in Tara Thompson's class have been working on the activity Half Yellow in the unit Finding Fair Shares. In prior activities in this unit, fractions are considered in relationship to one whole object, for example, one brownie or one hexagon cookie. As Ms. Thompson writes, "Before encountering Half Yellow, one half is one of two equal pieces of a brownie or the red trapezoid in a fraction cookie." In this activity, students' ideas about what half means are stretched. Students now must shift their view of what the whole is as they construct different combinations of pattern block shapes. As the "whole" changes to a group of pattern blocks instead of a single hexagon, students' ideas about "half" must also change. In this case, Ms. Thompson writes about her interactions with two students, Mary and Tamara, who have worked through this confusion and successfully made designs that are half yellow. When Ms. Thompson suggests they try the extension, One Third Yellow, they are once again challenged to shift their thinking.*

## Mary

Mary is a student with significant processing issues. It is difficult for her to make sense of and use language, especially when dealing with new or complex ideas. However, she has enjoyed the fraction work immensely. I think the visual nature of the activities has worked for her, and she has really been able to shine. She was excited when I suggested she make a design that was one third yellow.

When I approached Mary, she was finishing up coloring and labeling her design on paper. She had made a symmetrical design with a yellow hexagon, 6 green triangles, and 2 blues rhombuses.



**Teacher:** Can you show me where the thirds are?

**Mary:** Here's a yellow and six triangles is a part.

**Teacher:** And I see two blues. But I don't think your design is one third yellow. It is almost one third yellow. Can you figure out what to do to make it one third yellow? Can you show me the three equal parts?

Mary stares at her paper for a bit.

**Teacher:** You said it was one third yellow. So is yellow one of your equal parts?

She points to the yellow hexagon.

**Teacher:** What is another third of your design?

She points to some of the green triangles on her paper. Since I can't really tell what she is thinking, I wonder if rebuilding the design with blocks will help.

**Teacher:** Let's build it again. You used six greens, right?

I help her get the pieces she needs. She begins arranging the pieces.

**Teacher:** And two blue ones.

**Mary:** Yeah. . . . Wait, . . . Wait. Hold on.

I watch as she carefully replicates what she'd colored on the paper.

**Teacher:** Can you show me the three equal parts? Where are the thirds in your design?

I suggest that she put all the pieces together into hexagons. She puts all the greens together.

**Mary:** I need one more of these blue ones.

**Teacher:** Why do you need one more blue?

**Mary:** Because I have to make three wholes, and I only have two blues. And three blues make a whole.

She was busily rearranging the pieces into her original design.

**Mary:** Where will I put it?

I could see that the inclusion of this additional blue piece was going to ruin the symmetry of her design. I wondered if this helped cause the problem in the first place.

**Teacher:** Well, do you have to use a *blue* piece?

**Mary:** I could use two greens! They equal one blue.

And she happily found a place for them that kept the symmetry of her design.



## Tamara

Tamara is a good math thinker. She has great number sense and sees herself as a good math student. She, too, had been excited about the challenge of creating a design that was one third yellow. However, she came to me with a forlorn look, “I can’t do a third yellow.” First of all, she was trying to use several yellow parts in addition to several parts made of the other colors.

**Teacher:** What does thirds mean?

**Tamara:** Three parts.

**Teacher:** What needs to be true about those three parts?

**Tamara:** They have to be the same, equal.

I asked her to make a design that was all yellow, where she could see thirds. She put down three yellow hexagons.



**Teacher:** Why does this design have thirds? Why are these thirds?

**Tamara:** Because there are three parts and they’re equal.

**Teacher:** Where are the three equal parts?

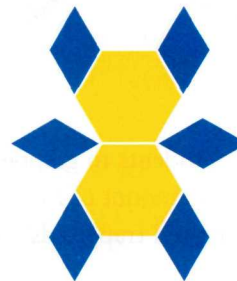
She points to each, one at a time.

**Teacher:** Where is one third? If you wanted to make a design that was only one *third* yellow, what would you have to do?

**Tamara:** Use other colors.

**Teacher:** Can you show me what you mean?

Another student needed me. When I came back, Tamara had a design made up of two yellows and six blues.



**Teacher:** Can you show me where your thirds are?

Tamara pointed to the two yellow parts and then began counting up the blues and putting them into hexagons.

**Tamara:** It isn’t thirds. . . . It’s fourths.

**Teacher:** Do you have to add to it or take something away?

I knew she really could have done either, but I wanted her to think about a design equal to three hexagons before she tried anything more complicated.

**Tamara:** I have to take a whole away.

**Teacher:** You have to take a whole away or do you need to take a hexagon away?

**Tamara (smiling):** A hexagon.

**Teacher:** So what are you taking away, something yellow or something blue?

**Tamara:** Something yellow.

She happily slides the yellow away and begins to copy her design onto the paper. But I was to see in other designs that she was still struggling with these expanded ideas about fractions and equivalency.



It takes a while for the students to grapple with these ideas. I plan on spending a good amount of time on this important activity. All halves aren't red trapezoids! All thirds aren't blue rhombuses!

Mary and Tamara are both working with a central idea in understanding fractions—that the fractional part is related to a particular whole. In work with fractions, students often start with a whole and divide it into fractional parts. These students are used to thinking of one of the pattern blocks as a whole and using it to show fractions. However, building up *from a part* ( $\frac{1}{3}$ ) to a whole provides a new challenge. In addition, the whole is now not just one pattern block but a design made up of a number of blocks. The teacher helps the students visualize the relationships of fractional part and whole by consistently referring to the pattern block

representation: “Can you show me where the thirds are?” By manipulating the representation, they can see if they do have three equal parts, one of which is yellow, and, with the teacher’s support, revise their representation.

### Questions for Discussion

1. What do students need to understand about fractions to make designs that are “half yellow” or “one third yellow?”
2. What do Mary and Tamara understand about thirds? What confuses them? What steps does Ms. Thompson take to help them work through their confusion?
3. What experiences have your students had with these activities? How have you helped them make the necessary shifts in their thinking about fractions?